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DYNAMIC GROWTH OF MICROVOIDS UNDER COMBINED HYDROSTATIC AND DEVIATORIC STRESSES

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Abstract—This paper deals with the analytical study of the dynamic plastic growth of microvoids under the combined action of hydrostatic and deviatoric stresses. The results of this analysis are discussed with the help of a numerical study of the void growth relationship derived, and applied to the case of spall fracture. The conclusion is that void expansion may be affected in different manners by the presence of a field of deviatoric (purely distortional) strain rates. If the deviatoric plastic strain rate is not large compared with the rate of volumetric expansion, then, for void growth controlled fracture, the spall strength of the material tends to decrease with respect to a purely hydrostatic stress. The quantitative loss of strength may be important, depending upon the loading conditions. When void growth initiates in a state of very large deviatoric strain rates then, under the conditions of the analysis, the volumetric expansion of the voids may require excessive large stresses, so as to become very difficult in practice. Then, in such a situation a different mechanism, such as void nucleation for instance, might control the fracture process rather than plastic void growth.

1. INTRODUCTION

Spall fracture is a type of fracture in the form of an internal cavitation, which is caused by the interaction of stress waves near a free surface. It has been identified that there are two fundamental modes of dynamic fracture: brittle, where spall is controlled by the evolution of microcracks in the material which propagate and finally coalesce to generate the spall plane, and ductile, where spall is controlled by the dynamic evolution and coalescence of microvoids, accompanied by a large plastic deformation of the material around the voids. Several attempts have been made to define the phenomena controlling spall fracture. It has been established that spall fracture is associated with the complicated concurrence of nucleation, growth and coalescence of microdefects, which can depend on both the pre-existing and the evolving microstructure. Review articles [see Curran (1982), Curran *et al.* (1987) and Meyers and Aimone (1983)] explain in some detail the most relevant results of both experimental and theoretical studies of spall fracture.

The spall strength of materials is a parameter which has been frequently found to vary in a wide range in different testing conditions, in such a way that no unique value of the spall strength can be associated to a given material (Meyers and Aimone, 1983; Grady, 1988; Buchar *et al.*, 1991; Zurek and Frantz, 1988). A plastic void growth model for ductile fracture such as those previously presented in Carroll and Holt (1972) and Johnson (1981), can be invoked to justify strain rate dependency of spall strength (Buchar *et al.*, 1991). In addition, careful examination of the fracture surfaces also show that a given material can suffer transition from ductile to brittle fracture (Grady, 1988), or from intergranular to transgranular fracture when the experimental conditions are varied (Buchar *et al.*, 1991; Zurek and Frantz, 1988), giving further arguments to justify the observed loading conditions dependency of the spall strength.

In the present paper, we investigate the influence that a combined state of hydrostatic and deviatoric stresses may have in the spall strength of ductile materials for the case of void growth controlled fracture. Although in the past several authors have studied the void growth problem under triaxiality conditions [see for instance Rice and Tracey (1968), Gurson (1977), Duva and Hutchinson (1984), Cocks (1989) and Ponte Castañeda and Willis (1988)] they have limited their analysis to static loading, ignoring the influence of inertial effects. In order to achieve our objective, we analyse in detail the plastic void growth process which takes place under extremely high rates of loading. The voids are assumed to be spherical and to grow plastically due to the action of a purely dilatational velocity field, plus a purely deviatoric velocity field, the latter causing shape distortion keeping the material volume constant. Then, we modify a procedure previously used by Gurson (1977) to derive the macroscopic yield locus of porous materials, in order to embody the inertia effects. Then, we derive the void growth equation under a combined stress state.

2. VOID GROWTH EQUATIONS

2.1. *General equations*

Expressions for macroscopic stresses are derived as a first step towards the void growth equations. In Gurson's paper, the actual distribution of voids in a matrix is replaced by a single spherical void embedded in a spherical portion of matrix material, an idea which, to the author's knowledge, was first introduced by Carroll and Holt (1972). The resulting thick hollow sphere is expected to represent in an average sense the actual behaviour of the porous material. Initially, the radius of the void is taken as a value representing the size of the actual voids, whereas the outer radius of the sphere is selected in order to agree with the initial volume fraction of voids present in the material. In the above model, each void was then assumed to be subjected to a velocity (or deformation) field composed of the sum of two different fields: the first one was a radial velocity field with spherical symmetry associated to a simple volumetric expansion of the hollow sphere, with no shape change to it. The strain velocity field components associated with such velocity field (subindexed with letter v) are given by:

$$\dot{\varepsilon}_{v,rr} = -\tfrac{2}{3}\dot{\varepsilon}_v (b/r)^3, \qquad \dot{\varepsilon}_{v,\theta\theta} = \dot{\varepsilon}_{v,\phi\phi} = -\dot{\varepsilon}_{v,rr}/2 \tag{1}$$

where $\dot{\varepsilon}_v$ is the macroscopic volumetric strain rate defined as $\dot{V}/V$, where V is the total macroscopic volume, indexes r, θ and ϕ indicate spherical coordinates components, and b is the outer radius of the void matrix model at the time considered. The value of the inner radius a of the void matrix model at a given instant is assumed to correspond to the actual void radius, whereas the ratio $(a/b)^3$ is assumed to be equal to the volume fraction of the voids ξ actually present in the material at the same instant.

The second one was a velocity field giving rise to pure distortion of the material keeping the total volume constant. So, we will refer to this latter velocity field as the deviatoric velocity field. This deviatoric velocity field was assumed to be uniform within the material, which may not be a rigorous assumption, but it is expected to represent, in a statistically averaged sense, the actual situation experienced by the material. In the present paper, and to avoid unnecessary complications, the components $\dot{e}_{ij}$ of this latter velocity field will be expressed in a system of principal axes of strain, and they will be assumed to be time independent.

Neglecting the inertia effects, Gurson (1977) was able to compute the macroscopic volumetric stress σ_V in a generic manner as:

$$\sigma_V = \frac{1}{V}(2/3)^{1/2} \int_V \sigma_e \, \partial(\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij})^{1/2} / \partial\dot{\varepsilon}_v \, \mathrm{d}V \tag{2}$$

where V is the macroscopic volume, σ_e is the microscopic equivalent yield stress, $\dot{\varepsilon}_V$ stands for the volumetric macroscopic deformation rate and $\dot{\varepsilon}_{ij}$ denotes the local strain rate tensor components. Then, for the particular velocity field selected, the macroscopic hydrostatic tension in equilibrium with the velocity field considered is:

$$\sigma_V = (2/3)^{1/2} \int_\xi^1 \sigma_e \frac{2\dot{\varepsilon}_V}{3} (3\dot{e}_D^2/2 + (2/3)\dot{\varepsilon}_V^2/\lambda^2)^{-1/2} \, \mathrm{d}\lambda/\lambda^2 \tag{3}$$

where $\lambda = (r/b)^3$, $\xi = (a/b)^3$ is the material porosity as before and $\dot{e}_D$ is the macroscopic deviatoric strain rate. $\dot{e}_D$ is defined as a function of the components $\dot{e}_i$ of the macroscopic deviatoric strain rate tensor in a system of principal axes of strain as $\dot{e}_D = (2\dot{e}_i\dot{e}_i/3)^{1/2}$.

For a perfectly plastic material for instance, where $\sigma_e = \sigma_0$, the macroscopic volumetric stress is, after integration:

$$\sigma_V = \frac{2\sigma_0}{3}\ln\left(\frac{(\dot{\varepsilon}_V^2+\xi^2(3\dot{e}_D/2)^2)^{1/2}+\dot{\varepsilon}_V}{\xi((\dot{\varepsilon}_V^2+(3\dot{e}_D/2)^2)^{1/2}+\dot{\varepsilon}_V)}\right). \quad (4)$$

This latter expression was previously derived by Gurson (1977).

2.2. *Void growth equations in the presence of deviatoric strain rates and inertia terms*

To include the inertia terms, Gurson's formulation is modified with the inclusion of the associated energy dissipation. We recall here that in Gurson's formulation the velocity field was assumed to be the sum of a radial velocity field with spherical symmetry, analogous to that of Carroll and Holt's paper, plus a purely deviatoric velocity field. By definition, this latter velocity field yielded only a distortion of the material keeping its macroscopic volume constant. From the principle of virtual work it follows that the macroscopic stresses σ_I equilibrating the inertia effects associated to volumetric expansion of the material will be given by the expression:

$$\sigma_I = \frac{\rho}{V}\int \mathbf{a}\cdot\frac{\partial \mathbf{r}}{\partial \varepsilon_V}\,dV \quad (5)$$

where ρ is mass density, $\mathbf{a}$ is the acceleration vector, $\mathbf{r}$ is the position vector and ε_V is the macroscopic volumetric strain. It is desirable for our purposes to eliminate from the above expression variables $\mathbf{r}$ and ε_V. To this end we remark that we are dealing with first order homogeneous velocity fields in the components of the macroscopic strain rates tensor. Then, it is accomplished that:

$$\mathbf{v} = \frac{\partial \mathbf{v}}{\partial \dot{\varepsilon}_V}\dot{\varepsilon}_V + \frac{\partial \mathbf{v}}{\partial \dot{e}_1}\dot{e}_1 + \frac{\partial \mathbf{v}}{\partial \dot{e}_2}\dot{e}_2 + \frac{\partial \mathbf{v}}{\partial \dot{e}_3}\dot{e}_3 \quad (6)$$

where $\mathbf{v}$ denotes the velocity vector. Then, by assuming that $\mathbf{r}$ is a function of ε_V and the e_i, it is easy to show that we must have:

$$\frac{\partial \mathbf{r}}{\partial \varepsilon_V} = \frac{\partial \mathbf{v}}{\partial \dot{\varepsilon}_V}. \quad (7)$$

In this way, eqn (5) can be expressed as:

$$\sigma_I = \frac{\rho}{V}\int \mathbf{a}\cdot\frac{\partial \mathbf{v}}{\partial \dot{\varepsilon}_V}\,dV. \quad (8)$$

As a consequence, expressions for velocities and accelerations are needed for evaluating σ_I. Since we know that, by definition, the velocity field associated with deviatoric strains is not dependent upon the volumetric strain rate $\dot{\varepsilon}_V$, it is clear that only the radial velocity field with spherical symmetry will contribute to the integral involved in eqn (8). Since these latter velocities are radial, we will need to compute only the radial components of the total acceleration, in order to evaluate the right-hand side of eqn (8). Moreover, the total acceleration is composed of two terms, namely, an acceleration associated with the radial velocity field with spherical symmetry, and a second term associated with the deviatoric velocity field. From all the above, the evaluation of the radial velocity and acceleration velocity fields with spherical symmetry has been made previously (Carroll and Holt, 1972;

Johnson, 1981). On the contrary, the evaluation of the radial component of the acceleration associated to the purely deviatoric velocity field needs some additional computation.

Subsequently, index i will be used to refer to the Cartesian components of velocities and accelerations, in principal axes of strain, associated with the deviatoric velocity field, whereas index R will indicate the radial component of the corresponding magnitudes. On the other hand, index r will refer to the components of velocity and acceleration of the radial velocity field with spherical symmetry analogous to that defined by Carroll and Holt.

From eqn (8) we can write:

$$\sigma_{\mathrm{I}} = \frac{\rho}{V} \int \left[a_{\mathrm{r}} \frac{\partial v_{\mathrm{r}}}{\partial \dot{\varepsilon}_{\mathrm{V}}} + a_{\mathrm{R}} \frac{\partial v_{\mathrm{r}}}{\partial \dot{\varepsilon}_{\mathrm{V}}} \right] \mathrm{d}V. \tag{9}$$

Both v_{r} and a_{r} are known from previous works (Carroll and Holt, 1972; Johnson, 1981), and are given by:

$$v_{\mathrm{r}} = -\dot{B}(t)/3r^2 \tag{10}$$

and

$$a_{\mathrm{r}} = \ddot{B}(t)/3r + \dot{B}(t)^2/18r^4 \tag{11}$$

where r is an internal radius and $B(t)$ is a function given by:

$$B(t) = a_0^3(\alpha_0 - \alpha)/(\alpha_0 - 1). \tag{12}$$

In the above equation, a_0 is the initial value of the void radius, α is the distention factor and α_0 is its corresponding initial value (Johnson, 1981). The distention factor is related to material porosity by the expression $\alpha = 1/(1-\xi)$.

We now evaluate a_{R}. It is important to remark at this stage that eqn (9) refers to the real values of velocities and accelerations. In our formulation, however, we are dealing with a uniform deviatoric strain rate field which, as previously said, represents material behaviour only in an average sense. So, since locally unbalanced inertia stresses may be introduced by the velocity fields considered, results obtained from these analyses should be taken to be valid in an average sense only.

The velocity field associated with deviatoric plastic strain rates is given by:

$$v_i = \dot{\mathrm{e}}_i x_i \tag{13}$$

where the x_i (i = 1, 2, 3) are Cartesian coordinates. The acceleration associated with this velocity field is:

$$a_i = \dot{\mathrm{e}}_i^2 x_i \tag{14}$$

and the radial component of the acceleration associated to the deviatoric velocity field will then be:

$$a_{\mathrm{R}} = r[\dot{\mathrm{e}}_1^2 \cos^2\theta + \dot{\mathrm{e}}_2^2 \operatorname{sen}^2\theta \cos^2\phi + \dot{\mathrm{e}}_3^2 \operatorname{sen}^2\theta \operatorname{sen}^2\phi] \tag{15}$$

where r, θ and ϕ are spherical coordinates. By replacing this latter expression into eqn (9) together with those corresponding to v_{r} and a_r, we finally obtain:

$$\sigma_{\mathrm{I}} = \frac{\rho a_0^2}{3(\alpha_0 - 1)^{2/3}} Q(\ddot{\alpha}, \dot{\alpha}, \alpha) + \rho \frac{\dot{\mathrm{e}}_{\mathrm{D}}^2}{4} (\alpha/(\alpha_0 - 1))^{2/3} a_0^2 (1 - \xi^{2/3}) \tag{16}$$

where the function $Q(\ddot{\alpha}, \dot{\alpha}, \alpha)$ is defined by eqn (17):

$$Q(\ddot{\alpha},\dot{\alpha},\alpha) = \ddot{\alpha}[(\alpha-1)^{-1/3}-\alpha^{-1/3}] - \frac{\dot{\alpha}^2}{6}[(\alpha-1)^{-4/3}-\alpha^{-4/3}]. \tag{17}$$

We recall here that $\alpha = 1/(1-\xi)$. Function Q has been expressed as a function of parameter α just to emphasize the fact that the term involving such a function in eqn (16) is simply the same inertia term associated with the growth of microvoids under a purely hydrostatic stress, an expression previously derived by Carroll and Holt (1972) and Johnson (1981). The term involving $\dot{e}_D^2$ in eqn (16) corresponds to the volumetric resistance introduced by the purely deviatoric velocity field. We see that this latter term always has a positive sign. This is due to the fact that a uniform deviatoric strain rate field has been assumed to be present and, consequently, particles increase their velocities as they move outwards [see eqn (13)], whereas they decrease their velocity as they move inward, in order to keep the strain rate constant. Thus, inertia forces associated with the deviatoric strain rate considered act inward, and a net positive power must be added in order to overcome such inertia forces.

So, from the above expressions the dynamic void growth equation becomes:

$$\sigma-\sigma_V = \frac{\rho a_0^2}{3(\alpha_0-1)^{2/3}}Q(\ddot{\alpha},\dot{\alpha},\alpha)+\rho\frac{\dot{e}_D^2}{4}(\alpha/(\alpha_0-1))^{2/3}a_0^2(1-\xi^{2/3}) \tag{18}$$

where σ is the macroscopic hydrostatic stress and σ_V is the plastic resistance to volumetric expansion.

2.3. *Influence of viscosity on dynamic void growth*

A more general expression considering the influence of a linear viscosity term of the type:

$$\sigma_e = \sigma_0(1+B\dot{\varepsilon}) \tag{19}$$

where σ_0 and B are constants, on the dynamic void growth relationships in presence of deviatoric plastic strain rates is now derived. Equation (19) is frequently found to fit with the experimental results in the very high strain rate regime.

It has been shown, Gurson (1977), that the local strain rate $\dot{\varepsilon}$ can be expressed as:

$$\dot{\varepsilon}^2 = \tfrac{2}{3}(\dot{e}_i\dot{e}_i+\tfrac{2}{3}(b/r)^6\dot{\varepsilon}_v^2-2(b/r)^3\dot{\varepsilon}_v\dot{e}_r) \tag{20}$$

where $\dot{e}_r$ is the radial component of tensor $\dot{e}_i$. Then, by developing into series to the first order in $\dot{e}_r$, it is obtained by substitution of eqn (19) into eqn (3) that the macroscopic volumetric stress equilibrating the plastic resistance to deformation is:

$$\sigma_V = \frac{2\sigma_0}{3}\ln\left(\frac{(\dot{\varepsilon}_V^2+\xi^2(3\dot{e}_D/2)^2)^{1/2}+\dot{\varepsilon}_V}{\xi((\dot{\varepsilon}_V^2+(3\dot{e}_D/2)^2)^{1/2}+\dot{\varepsilon}_V)}\right)+(2/3)^2\sigma_0 B\dot{\varepsilon}_V\frac{1-\xi}{\xi}. \tag{21}$$

Consequently, the dynamic void growth equation for this viscous material is eqn (18), where σ_V is given by eqn (21). Expressions for σ_V corresponding to more complicated viscous behaviour have been derived elsewhere (Cortés, 1989).

3. NUMERICAL ANALYSIS

In this section, we study numerically the analytical formulation previously developed, assuming a material subjected to a linearly increasing hydrostatic pressure at a rate of 1 GPa μs^{-1} and 10 GPA μs^{-1}, alternatively. These are loading rates which may be typically found in impact or explosive loading situations.

For this analysis we have considered a rigid–perfectly plastic material, and we have chosen parameter values of $\sigma_0 = 150$ MPa and $\rho = 8930$ kg m^{-3}, which may correspond to copper. A value of $a_0 = 10^{-6}$ m for the initial pore size was selected, as well as an initial porosity of $\xi_0 = 10^{-4}$.

The evolution of porosity with time was obtained from the numerical integration of the dynamic void growth equation, a mechanism which was also assumed to control fracture. It is well known that coalescence of cavities takes place at a given instant as voids grow. In this simulation, and just to fix ideas, it is assumed that coalescence takes place by direct impingement of the cavities when the distance between cavities equals the void radius. This implies that the corresponding porosity at the instant of coalescence equals $\xi = 0.30$. In consequence, the dynamic tensile strength is defined as the hydrostatic stress acting on the material for a porosity value of $\xi = 0.30$, and the numerical analyses were made up to the moment when such a value was reached. In the case of copper, this choice agrees with experimental observations (Perzyna, 1986).

The influence of deviatoric macroscopic strain rates on the dynamic porosity curves for a perfectly plastic material is shown in Fig. 1. In this case values of the macroscopic deviatoric strain rate $\dot{e}_D$ of 0, 10^3 s^{-1}, 10^4 s^{-1}, 10^5 s^{-1} and 10^6 s^{-1} were selected, and a hydrostatic loading rate of 1.0 GPa μs^{-1} was superimposed for all such cases. It is clearly seen in the figure that the softening effect of the deviatoric field, which causes a decrease in the hydrostatic stress for a given porosity value as the deviatoric strain rate $\dot{e}_D$ increases. Moreover, it can also be seen that the value of the tensile strength is not appreciably affected by the selection of the critical value of porosity (here taken as $\xi = 0.3$), provided that such a critical value exceeds a small value ranging from 10^{-4} when $\dot{e}_D = 0$, to about 10^{-2}, when $\dot{e}_D = 10^6$ s^{-1}. In effect, for larger porosity values the slope of the stress–porosity curve tends to be very small, and the stress increases very little until fracture finally takes place. Since a constant loading rate of 1 GPa μs^{-1} has been imposed, time–porosity curves can be directly obtained from Fig. 1, without making any change in the figure apart from reading μs in the vertical scale rather than GPa. Then, by taking into account that a simple calculation shows that $\dot{\varepsilon}_V = \dot{\xi}/(1-\xi)$ (Cortés, 1989), the volumetric strain histories corresponding to Fig. 1 may be easily estimated. In Fig. 2 the dynamic tensile strengths corresponding to hydrostatic loading rates of 1 GPa μs^{-1} and 10 GPa μs^{-1}, alternatively, are plotted as a function of the macroscopic deviatoric strain rate. In this figure it can be observed that the tensile strength remains nearly unaffected by the value of $\dot{e}_D$, for values of this parameter up to about 5×10^2 s^{-1} for a loading rate of 1 GPa μs^{-1}, and up to about 10^4 s^{-1} for a loading rate of 10 GPa μs^{-1}. For higher values, the dynamic strength–deviatoric strain rate curve has a negative slope and, consequently, the dynamic tensile strength decreases as the deviatoric strain rate increases. This part of the curve corresponds to a situation where the effect of the deviatoric strain rate field has a net softening effect of the material, since it supports part of the energy required to cause plastic yielding. Both

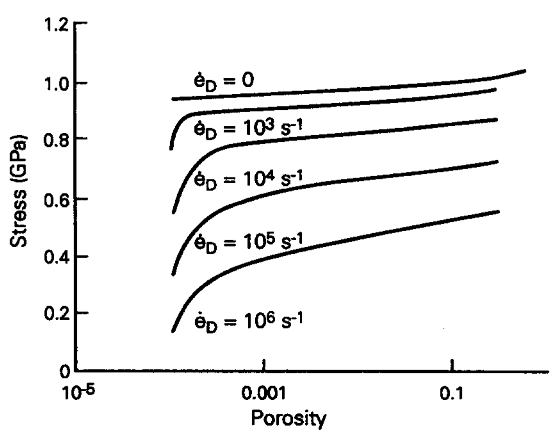

Fig. 1. Stress–porosity curves for $\dot{\sigma} = 1$ GPa μs^{-1} and the indicated values of the deviatoric strain rate.

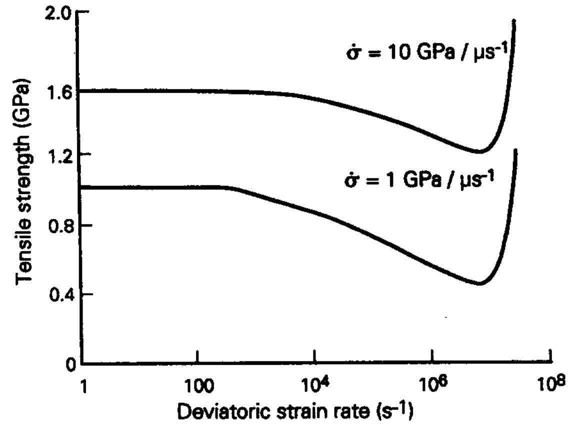

Fig. 2. Dynamic tensile strength as a function of the macroscopic deviatoric strain rate for $\dot{\sigma} = 1$ GPa μs^{-1} and $\dot{\sigma} = 10$ GPa μs^{-1}.

curves shown in Fig. 2 show an absolute minimum at a value of the deviatoric strain rate of about 6×10^6 s^{-1}. We can also see that the decrease in the tensile strength for such a value of the deviatoric strain rate is very important for the case of a hydrostatic loading rate of 1 GPa μs^{-1}, where the dynamic strength reduces to only 44% of the corresponding value under a purely hydrostatic loading. At a loading rate of 10 GPa μs^{-1} this reduction is less marked, and the value of the dynamic strength is about 77% of the value obtained when the deviatoric strain rate vanishes. For higher values the dynamic tensile strength increases very rapidly. This phenomenon corresponds to a situation where the energy required to overcome the inertia effects introduced by the deviatoric strain rate field exceeds the corresponding softening effect, in such a manner that plastic void growth becomes increasingly difficult.

4. DISCUSSION

We have seen in the previous section that the presence of macroscopic deviatoric strain rates is a factor which may have a remarkable influence on the dynamic tensile strength of the material. Although in practice it is virtually impossible to control the hydrostatic and the deviatoric components of the strain rate field which will be experienced by the material in a given test, it is implied by the present work that, for void growth controlled fracture, the value of the tensile strength may be affected by the particular deformation velocity field under which fracture takes place. This fact agrees with the experimental observations, in the sense that spall strength variations over wide ranges are usually observed for a given material. From results shown in Fig. 2, it appears that spall fracture controlled by ductile void growth may take place whenever the deviatoric strain rate does not exceed a sufficiently large value. Otherwise, the hydrostatic stress needed for void growth becomes excessively large, and it is expected in such conditions that a different mechanism requiring lower stresses than those predicted by Fig. 2, such as void nucleation for instance, should instead control spall fracture. Naturally, since from Fig. 2 we see that the dynamic strength also increases with the value of the hydrostatic loading rate, a similar conclusion could be stated referring to increasing values of such parameter. In fact, it has been observed for copper that a competitive growth process between voids inside the grains, where a tendency to void nucleation is observed, and those at grain boundaries seems to take place under impact loading (Buchar *et al.*, 1991). So, for strain rates lower than a critical value the spall strength is markedly strain rate dependent, and fracture tends to be intergranular. On the contrary, for higher deformation velocities the spall strength becomes insensitive to strain rate, and dynamic fracture comes to be transgranular (Buchar *et al.*, 1991). So, it is clear that the strain rate sensitive behaviour associated with intergranular fracture can be explained by a plastic void growth model, analogous to that presented in this work, which predicts that both the hydrostatic loading rate and the deviatoric strain rate have influence in the material tensile strength. For higher strain rates (either hydrostatic or deviatoric), we have seen that inertia effects become larger making increasingly difficult plastic void growth, and so, fracture initiation might have to be favoured at void nucleation sites. Consequently, according to the present model one might expect a different behaviour in such conditions. Furthermore, it is frequently found that there is an exponential dependence of the nucleation rate upon the applied stress (Curran *et al.*, 1987). This means that shortly after the stress for the onset of nucleation is surpassed, the nucleation rate becomes important and may eventually control the microstructural damage and thus, material fracture. So, in such conditions, the spall strength would be expected to vary very little with the loading rate, in qualitative agreement with the experimental observations for copper of a strain rate insensitive spall strength. Naturally, in such cases the curves shown in Fig. 2 should deviate, in the range of the highest strain rates when a sudden increase in the tensile strength is predicted, to lower stresses.

The value of the deviatoric strain rate at which the minimum dynamic tensile strength occurs (see Fig. 2), is a value which is chiefly affected by the initial microstructural parameters. In effect, we can roughly estimate the value of the critical deviatoric strain rate, that is, the value of $\dot{e}_D$, above which inertia effects become important, by equating the second term of the right-hand side of eqn (18) to an appropriate fraction of the yield point of the material, say, $f\sigma_0$, being σ_0 the elastic limit and f a factor much smaller than unity. Initially, ξ is very close to zero, and thus α can be approximated by $1+\xi$. Then, by recalling that $\xi = (a/b)^3$, we can state the condition for the onset of inertia controlled void growth as $\dot{e}_D^2 = 4f\sigma_0/(\rho b_0^2)$, where b_0 is one half of the distance between voids. Since it is recognized that voids may nucleate at inclusions and second phase particles, the result is that, if the material purity is increased, the value of b_0 will also increase. Then, the critical value of $\dot{e}_D$ will rapidly decrease and, consequently, plastic controlled void growth may become difficult at lower deviatoric strain rates, thus favouring the appearance of a different controlling mechanism for ductile fracture, such as void nucleation for instance. This result may explain, at least qualitatively, why transition from intergranular to transgranular fracture is a phenomenon observed only when copper is of relatively high purity (Buchar *et al.*, 1991).

5. CONCLUSIONS

In this work has been estimated the effect that the interaction of a purely hydrostatic field and a purely deviatoric stress field, has on the spall strength of ductile materials for void growth controlled fracture. It has been discovered that, when the volumetric expansion rate is more important than the deviatoric strain rate then, for void growth controlled dynamic fracture, the spall strength may be affected by the deviatoric field and, under certain circumstances, cause a considerable decrease in the strength of the material. On the contrary, when the deviatoric strain rate is very high, stresses required to overcome the inertia effects associated with the deviatoric strain rate field may become very large, in such a way that plastic void growth may be very difficult. Under such conditions, it is expected that a different mechanism, such as void nucleation for instance, should be the controlling factor in dynamic fracture.

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